In these problems draw (using Manipulate) a point moving on each curve and display the curvature of the curve at the point by means of Tooltip.

1. Graph the curvatures of the cycloids illustrated in Figures 2.4 and 2.5. Find the formula for the curvature $\boldsymbol{\kappa}\mathbf{2}$ of the general cycloid cycloid$[a, b]$. Then define and draw ordinary, prolate and curtate cycloids together with the defining circle such as those on page 42.

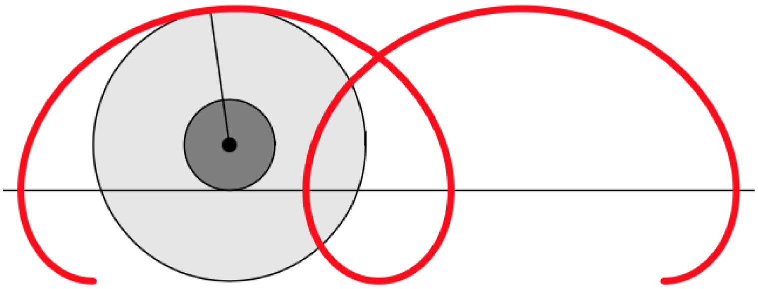

Figure 2.4: The prolate cycloid $t \mapsto (t - 3\sin t,\ 1 - 3\cos t)$

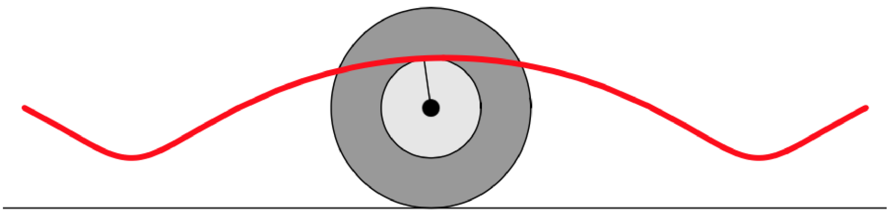

Figure 2.5: The curtate cycloid $t \mapsto (2t - \sin t,\ 2 - \cos t)$

2. A ***deltoid*** is defined by

$$\text{deltoid}[a](t) = \big(2a\cos t(1 + \cos t) - a,\ 2a\sin t(1 - \cos t)\big).$$

The curve is so named because it resembles a Greek capital delta. It is a particular case of a curve called ***hypocycloid*** (see Exercise 13 of Chapter 6). Plot as one graph the deltoids deltoid$[a]$ for $a = 1, 2, 3, 4$. Graph the curvature of the first deltoid.

3. The ***Lissajous***[7] or ***Bowditch curve***[8] is defined by

$$\text{lissajous}[n, d, a, b](t) = \big(a\sin(nt + d),\ b\sin t\big).$$

Draw several of these curves and plot their curvatures. (One is shown in Figure 11.19 on page 349.)

4. The ***limaçon***, sometimes called ***Pascal's snail***, named after Étienne Pascal, father of Blaise Pascal[9], is a generalization of the cardioid. It is defined by

$$\mathsf{limacon}[a,b](t) = (2a\cos t + b)\big(\cos t,\ \sin t\big).$$

Find the formula for the curvature of the limaçon, and plot several of them.

6. Define the curve

$$\mathsf{tschirnhausen}[n,a](t) = \left(a\frac{\cos t}{(\cos(t/3))^n},\ a\frac{\sin t}{(\cos(t/3))^n}\right).$$

When $n = 1$, this curve is attributed to Tschirnhausen[11]. Find the formula for the curvature of $\mathsf{tschirnhausen}[n,a][t]$ and make a simultaneous plot of the curves for $1 \leqslant n \leqslant 8$.